# $4^{\text {th }}$ SEMESTER 

RITU MADAM

## MAXIMIZATION OF TAX REVENUE

When govt imposes exicise duty on producers
WHAT DOES A PRODUCER DO ?
It wishes to maximize the tax revenue
HOW?
What should be the rate of $\operatorname{tax}(\mathrm{t})$ that the govt should choose to ensure that the tax revenue $(\mathrm{T})$ is maximised

2 different cases of maximization
(i) Imperfect competition
(ii) Competitive Market

IMPERFECT COMPETITION
Suppose for a firm
$T R=\alpha q-\beta q^{2}$
$(\alpha, \beta>0)$
$T C=a q^{2}+b q+c$
$(a, b, c>0)$
After imposition of cost
$C^{*}=C+t q$
$C^{*}=a q^{2}+b q+c+t q$
Profit function
$\Pi=\mathrm{R}-\mathrm{C}^{*}$
$\Pi=\alpha q-\beta q^{2}-a q^{2}-b q-c-t q \beta$
$1^{\text {st }}$ order condition for profit maximization
$\mathrm{d} \Pi / \mathrm{dq}=0$
$2^{\text {nd }}$ order condition
$\mathrm{d}^{2} \Pi / \mathrm{dq}^{2}>0$
$d \Pi / d q=a--2 \beta q-2 a q-b-t=0$
$\qquad$
$\qquad$
$q=a-b-t / 2(\beta+a)$
Now

$$
d^{2} \Pi / d q^{2}=--2 \beta-2 a<0 . . . . .(\text { from } i)
$$

Hence as both first and second order conditions are fulfilled
$q^{*}($ equibrium output) $=a--b-t / 2(\beta+a)$
TOTAL TAX REVENUE ( T ) = tq*
Hence $T=(a-b) t-t^{2} / 2(\beta+a)$ $\qquad$
NOW
Maximization of TAX REVENUE requires the fulfilment of first and second order condition
The question that arises now is what shoud be the tax rate that will maximize TAX REVENUE
First order condition $d T / d t^{2}=0$
From (ii) which gives $\quad(a-b)-2 t / 2(\beta+a)=0$

$$
t=a-b / 2
$$

Second order condition $d^{2} T / d t^{2}=--2 / 2(\beta+a)<0$

$$
\text { Hence } t=a-b / 2 \text { maximizes tax revenue }
$$

(1) A monopolist has TR and TC function as

$$
\begin{aligned}
& R=46 q-3 q^{2} \\
& C=2 q^{2}-4 q+10 \quad \text { where } q=\text { quantity produced }
\end{aligned}
$$

An excise tax at rate ' $t$ ' is imposed on output. Find
(i) Tax rate which will maximize total excise revenue of the govt
(ii) Monopolist maximum profit after payment of tax and profit maximizing output and the price at which product is sold

Sol ${ }^{n} \quad C^{*}=\mathrm{C}+\mathrm{tq}$

$$
\Pi=\mathrm{R}-\mathrm{C}^{*}
$$

$$
\begin{equation*}
\Pi=\mathrm{R}-\mathrm{C}-\mathrm{tq} \tag{i}
\end{equation*}
$$

Or $\quad \Pi=46 q-3 q^{2}-2 q^{2}+4 q-10-t q$
Maximisation of profit after tax requires the fulfilment of the following:

First order condition, $\mathrm{d} \Pi / \mathrm{dq}=0$ and
Second order condition, $\quad d^{2} \Pi / d q^{2}<0$
Now $d \Pi / d q=0$ gives,

$$
46-6 q-4 q+4-t=0
$$

Or

$$
\begin{array}{r}
10 q=50-t \\
50-t
\end{array}
$$

q $=10$

The excise tax revenue is given by

Or

$$
=50 t-t^{2} / 10
$$

Maximisation of $T$ requires $d T / d t=0$ and $d^{2} T / \mathrm{dt}^{2}<0$

Now, dT/dt = 0 gives,

$$
50-2 t / 10=0
$$

Or
Or

$$
\begin{aligned}
& 2 t=50 \\
& t=25
\end{aligned}
$$

Again, $\mathrm{d}^{2} \Pi / \mathrm{dt}^{2}=-2 / 10<0$
Therefore $t=25$ maximises total tax revenue

